1.5 Rewriting Equations and Formulas

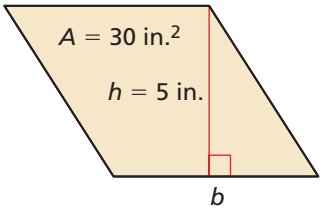
Essential Question How can you use a formula for one measurement to write a formula for a different measurement?

EXPLORATION 1 Using an Area Formula

Work with a partner.

a. Write a formula for the area A of a parallelogram.

b. Substitute the given values into the formula. Then solve the equation for b. Justify each step.



c. Solve the formula in part (a) for b without first substituting values into the formula. Justify each step.

d. Compare how you solved the equations in parts (b) and (c). How are the processes similar? How are they different?

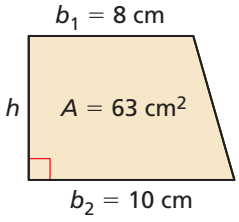
REASONING QUANTITATIVELY

To be proficient in math, you need to consider the given units. For instance, in Exploration 1, the area A is given in square inches and the height h is given in inches. A unit analysis shows that the units for the base b are also inches, which makes sense.

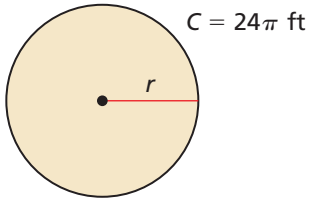
EXPLORATION 2 Using Area, Circumference, and Volume Formulas

Work with a partner. Write the indicated formula for each figure. Then write a new formula by solving for the variable whose value is not given. Use the new formula to find the value of the variable.

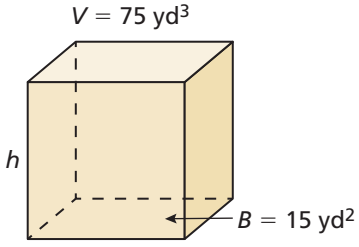
a. Area A of a trapezoid



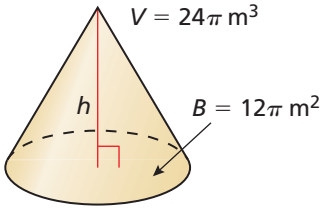
b. Circumference C of a circle



c. Volume V of a rectangular prism



d. Volume V of a cone



Communicate Your Answer

3. How can you use a formula for one measurement to write a formula for a different measurement? Give an example that is different from those given in Explorations 1 and 2.

1.5 Lesson

Core Vocabulary

literal equation, *p. 36*
formula, *p. 37*

Previous
surface area

What You Will Learn

- Rewrite literal equations.
- Rewrite and use formulas for area.
- Rewrite and use other common formulas.

Rewriting Literal Equations

An equation that has two or more variables is called a **literal equation**. To rewrite a literal equation, solve for one variable in terms of the other variable(s).

EXAMPLE 1 Rewriting a Literal Equation

Solve the literal equation $3y + 4x = 9$ for y.

SOLUTION

$3y + 4x = 9$ — Write the equation.

$3y + 4x - 4x = 9 - 4x$ — Subtract $4x$ from each side.

$3y = 9 - 4x$ — Simplify.

$\frac{3y}{3} = \frac{9 - 4x}{3}$ — Divide each side by 3.

$y = 3 - \frac{4}{3}x$ — Simplify.

▶ The rewritten literal equation is $y = 3 - \frac{4}{3}x$.

EXAMPLE 2 Rewriting a Literal Equation

Solve the literal equation $y = 3x + 5xz$ for x.

SOLUTION

$y = 3x + 5xz$ — Write the equation.

$y = x(3 + 5z)$ — Distributive Property

$\frac{y}{3 + 5z} = \frac{x(3 + 5z)}{3 + 5z}$ — Divide each side by $3 + 5z$.

$\frac{y}{3 + 5z} = x$ — Simplify.

▶ The rewritten literal equation is $x = \frac{y}{3 + 5z}$.

REMEMBER
Division by 0 is undefined.

In Example 2, you must assume that $z \neq -\frac{3}{5}$ in order to divide by $3 + 5z$. In general, if you have to divide by a variable or variable expression when solving a literal equation, you should assume that the variable or variable expression does not equal 0.

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Solve the literal equation for y.

1. $3y - x = 9$ **2.** $2x - 2y = 5$ **3.** $20 = 8x + 4y$

Solve the literal equation for x.

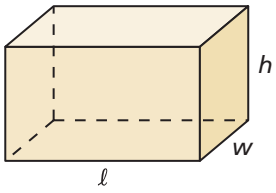
4. $y = 5x - 4x$ **5.** $2x + kx = m$ **6.** $3 + 5x - kx = y$

Rewriting and Using Formulas for Area

A **formula** shows how one variable is related to one or more other variables. A formula is a type of literal equation.

EXAMPLE 3 Rewriting a Formula for Surface Area

The formula for the surface area S of a rectangular prism is $S = 2\ell w + 2\ell h + 2wh$. Solve the formula for the length ℓ.



SOLUTION

$$S = 2\ell w + 2\ell h + 2wh$$ Write the equation.

$$S - 2wh = 2\ell w + 2\ell h + 2wh - 2wh$$ Subtract $2wh$ from each side.

$$S - 2wh = 2\ell w + 2\ell h$$ Simplify.

$$S - 2wh = \ell(2w + 2h)$$ Distributive Property

$$\frac{S - 2wh}{2w + 2h} = \frac{\ell(2w + 2h)}{2w + 2h}$$ Divide each side by $2w + 2h$.

$$\frac{S - 2wh}{2w + 2h} = \ell$$ Simplify.

▶ When you solve the formula for ℓ, you obtain $\ell = \frac{S - 2wh}{2w + 2h}$.

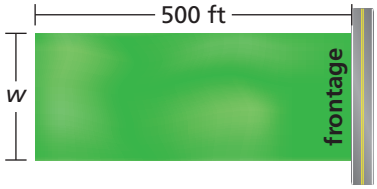
EXAMPLE 4 Using a Formula for Area

You own a rectangular lot that is 500 feet deep. It has an area of 100,000 square feet. To pay for a new water system, you are assessed $5.50 per foot of lot frontage.

a. Find the frontage of your lot.

b. How much are you assessed for the new water system?

SOLUTION



a. In the formula for the area of a rectangle, let the width w represent the lot frontage.

$$A = \ell w$$ Write the formula for area of a rectangle.

$$\frac{A}{\ell} = w$$ Divide each side by ℓ to solve for w.

$$\frac{100{,}000}{500} = w$$ Substitute 100,000 for A and 500 for ℓ.

$$200 = w$$ Simplify.

▶ The frontage of your lot is 200 feet.

b. Each foot of frontage costs $5.50, and $\frac{\$5.50}{1\ \cancel{ft}} \cdot 200\ \cancel{ft} = \$1100$.

▶ So, your total assessment is $1100.

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Solve the formula for the indicated variable.

7. Area of a triangle: $A = \frac{1}{2}bh$; Solve for h.

8. Surface area of a cone: $S = \pi r^2 + \pi r\ell$; Solve for ℓ.

Rewriting and Using Other Common Formulas

Core Concept

Common Formulas

Temperature	F = degrees Fahrenheit, C = degrees Celsius $C = \frac{5}{9}(F - 32)$
Simple Interest	I = interest, P = principal, r = annual interest rate (decimal form), t = time (years) $I = Prt$
Distance	d = distance traveled, r = rate, t = time $d = rt$

EXAMPLE 5 Rewriting the Formula for Temperature

Solve the temperature formula for F.

SOLUTION

$C = \frac{5}{9}(F - 32)$ Write the temperature formula.

$\frac{9}{5}C = F - 32$ Multiply each side by $\frac{9}{5}$.

$\frac{9}{5}C + 32 = F - 32 + 32$ Add 32 to each side.

$\frac{9}{5}C + 32 = F$ Simplify.

▶ The rewritten formula is $F = \frac{9}{5}C + 32$.

EXAMPLE 6 Using the Formula for Temperature

Which has the greater surface temperature: Mercury or Venus?

SOLUTION

Convert the Celsius temperature of Mercury to degrees Fahrenheit.

$F = \frac{9}{5}C + 32$ Write the rewritten formula from Example 5.

$= \frac{9}{5}(427) + 32$ Substitute 427 for C.

$= 800.6$ Simplify.

▶ Because 864°F is greater than 800.6°F, Venus has the greater surface temperature.

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9. A fever is generally considered to be a body temperature greater than 100°F. Your friend has a temperature of 37°C. Does your friend have a fever?

EXAMPLE 7 Using the Formula for Simple Interest

You deposit $5000 in an account that earns simple interest. After 6 months, the account earns $162.50 in interest. What is the annual interest rate?

SOLUTION

To find the annual interest rate, solve the simple interest formula for r.

$I = Prt$ Write the simple interest formula.

$\frac{I}{Pt} = r$ Divide each side by Pt to solve for r.

$\frac{162.50}{(5000)(0.5)} = r$ Substitute 162.50 for I, 5000 for P, and 0.5 for t.

$0.065 = r$ Simplify.

▶ The annual interest rate is 0.065, or 6.5%.

COMMON ERROR

The unit of t is years. Be sure to convert months to years.

$\frac{1 \text{ yr}}{12 \cancel{\text{mo}}} \cdot 6 \cancel{\text{mo}} = 0.5 \text{ yr}$

EXAMPLE 8 Solving a Real-Life Problem

A truck driver averages 60 miles per hour while delivering freight to a customer. On the return trip, the driver averages 50 miles per hour due to construction. The total driving time is 6.6 hours. How long does each trip take?

SOLUTION

Step 1 Rewrite the Distance Formula to write expressions that represent the two trip times. Solving the formula $d = rt$ for t, you obtain $t = \frac{d}{r}$. So, $\frac{d}{60}$ represents the delivery time, and $\frac{d}{50}$ represents the return trip time.

Step 2 Use these expressions and the total driving time to write and solve an equation to find the distance one way.

$\frac{d}{60} + \frac{d}{50} = 6.6$ The sum of the two trip times is 6.6 hours.

$\frac{11d}{300} = 6.6$ Add the left side using the LCD.

$11d = 1980$ Multiply each side by 300 and simplify.

$d = 180$ Divide each side by 11 and simplify.

The distance one way is 180 miles.

Step 3 Use the expressions from Step 1 to find the two trip times.

▶ So, the delivery takes $180 \text{ mi} \div \frac{60 \text{ mi}}{1 \text{ h}} = 3$ hours, and the return trip takes $180 \text{ mi} \div \frac{50 \text{ mi}}{1 \text{ h}} = 3.6$ hours.

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10. How much money must you deposit in a simple interest account to earn $500 in interest in 5 years at 4% annual interest?

11. A truck driver averages 60 miles per hour while delivering freight and 45 miles per hour on the return trip. The total driving time is 7 hours. How long does each trip take?

1.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. VOCABULARY Is $9r + 16 = \frac{\pi}{5}$ a literal equation? Explain.

2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Solve $3x + 6y = 24$ for x.

Solve $24 - 3x = 6y$ for x.

Solve $6y = 24 - 3x$ for y in terms of x.

Solve $24 - 6y = 3x$ for x in terms of y.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the literal equation for y. *(See Example 1.)*

3. $y - 3x = 13$

4. $2x + y = 7$

5. $2y - 18x = -26$

6. $20x + 5y = 15$

7. $9x - y = 45$

8. $6x - 3y = -6$

9. $4x - 5 = 7 + 4y$

10. $16x + 9 = 9y - 2x$

11. $2 + \frac{1}{6}y = 3x + 4$

12. $11 - \frac{1}{2}y = 3 + 6x$

In Exercises 13–22, solve the literal equation for x. *(See Example 2.)*

13. $y = 4x + 8x$

14. $m = 10x - x$

15. $a = 2x + 6xz$

16. $y = 3bx - 7x$

17. $y = 4x + rx + 6$

18. $z = 8 + 6x - px$

19. $sx + tx = r$

20. $a = bx + cx + d$

21. $12 - 5x - 4kx = y$

22. $x - 9 + 2wx = y$

23. MODELING WITH MATHEMATICS The total cost C (in dollars) to participate in a ski club is given by the literal equation $C = 85x + 60$, where x is the number of ski trips you take.

a. Solve the equation for x.

b. How many ski trips do you take if you spend a total of \$315? \$485?

24. MODELING WITH MATHEMATICS The penny size of a nail indicates the length of the nail. The penny size d is given by the literal equation $d = 4n - 2$, where n is the length (in inches) of the nail.

a. Solve the equation for n.

b. Use the equation from part (a) to find the lengths of nails with the following penny sizes: 3, 6, and 10.

ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in solving the equation for x.**

25.

$$12 - 2x = -2(y - x)$$
$$-2x = -2(y - x) - 12$$
$$x = (y - x) + 6$$

26.

$$10 = ax - 3b$$
$$10 = x(a - 3b)$$
$$\frac{10}{a - 3b} = x$$

In Exercises 27–30, solve the formula for the indicated variable. *(See Examples 3 and 5.)*

27. Profit: $P = R - C$; Solve for C.

28. Surface area of a cylinder: $S = 2\pi r^2 + 2\pi rh$; Solve for h.

29. Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$; Solve for b_2.

30. Average acceleration of an object: $a = \frac{v_1 - v_0}{t}$; Solve for v_1.

31. REWRITING A FORMULA A common statistic used in professional football is the quarterback rating. This rating is made up of four major factors. One factor is the completion rating given by the formula

$$R = 5\left(\frac{C}{A} - 0.3\right)$$

where C is the number of completed passes and A is the number of attempted passes. Solve the formula for C.

32. REWRITING A FORMULA Newton's law of gravitation is given by the formula

$$F = G\left(\frac{m_1 m_2}{d^2}\right)$$

where F is the force between two objects of masses m_1 and m_2, G is the gravitational constant, and d is the distance between the two objects. Solve the formula for m_1.

33. MODELING WITH MATHEMATICS The sale price S (in dollars) of an item is given by the formula $S = L - rL$, where L is the list price (in dollars) and r is the discount rate (in decimal form). *(See Examples 4 and 6.)*

a. Solve the formula for r.

b. The list price of the shirt is \$30. What is the discount rate?

34. MODELING WITH MATHEMATICS The density d of a substance is given by the formula $d = \frac{m}{V}$, where m is its mass and V is its volume.

Pyrite

Density: 5.01g/cm^3 Volume: 1.2 cm^3

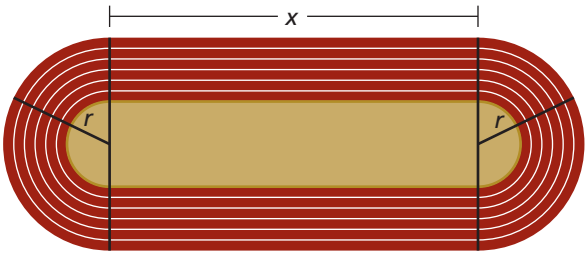
a. Solve the formula for m.

b. Find the mass of the pyrite sample.

35. PROBLEM SOLVING You deposit \$2000 in an account that earns simple interest at an annual rate of 4%. How long must you leave the money in the account to earn \$500 in interest? *(See Example 7.)*

36. PROBLEM SOLVING A flight averages 460 miles per hour. The return flight averages 500 miles per hour due to a tailwind. The total flying time is 4.8 hours. How long is each flight? Explain. *(See Example 8.)*

37. USING STRUCTURE An athletic facility is building an indoor track. The track is composed of a rectangle and two semicircles, as shown.

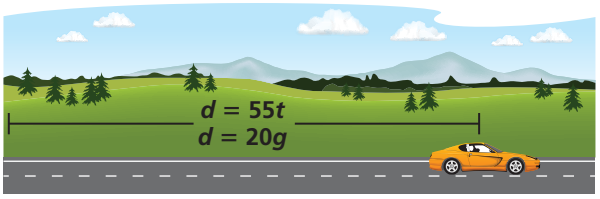


a. Write a formula for the perimeter of the indoor track.

b. Solve the formula for x.

c. The perimeter of the track is 660 feet, and r is 50 feet. Find x. Round your answer to the nearest foot.

38. MODELING WITH MATHEMATICS The distance d (in miles) you travel in a car is given by the two equations shown, where t is the time (in hours) and g is the number of gallons of gasoline the car uses.

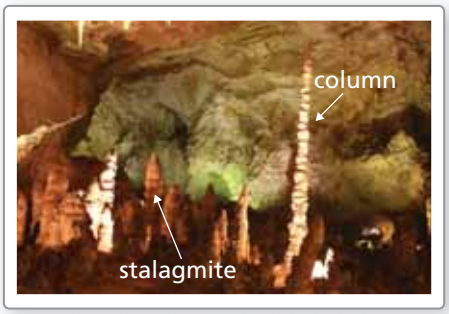


a. Write an equation that relates g and t.

b. Solve the equation for g.

c. You travel for 6 hours. How many gallons of gasoline does the car use? How far do you travel? Explain.

39. MODELING WITH MATHEMATICS One type of stone formation found in Carlsbad Caverns in New Mexico is called a column. This cylindrical stone formation connects to the ceiling and the floor of a cave.

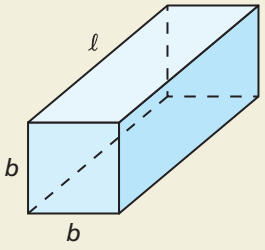


a. Rewrite the formula for the circumference of a circle, so that you can easily calculate the radius of a column given its circumference.

b. What is the radius (to the nearest tenth of a foot) of a column that has a circumference of 7 feet? 8 feet? 9 feet?

c. Explain how you can find the area of a cross section of a column when you know its circumference.

40. HOW DO YOU SEE IT? The rectangular prism shown has bases with equal side lengths.

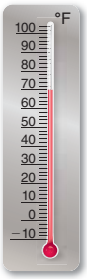


a. Use the figure to write a formula for the surface area S of the rectangular prism.

b. Your teacher asks you to rewrite the formula by solving for one of the side lengths, b or ℓ. Which side length would you choose? Explain your reasoning.

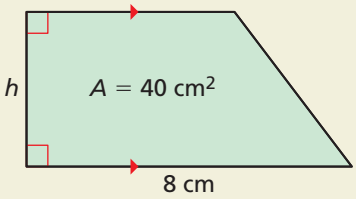
41. MAKING AN ARGUMENT Your friend claims that Thermometer A displays a greater temperature than Thermometer B. Is your friend correct? Explain your reasoning.

Thermometer A



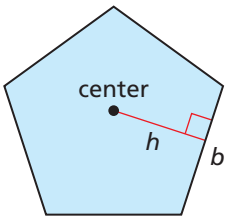
Thermometer B

42. THOUGHT PROVOKING Give a possible value for h. Justify your answer. Draw and label the figure using your chosen value of h.



MATHEMATICAL CONNECTIONS In Exercises 43 and 44, write a formula for the area of the regular polygon. Solve the formula for the height h.

43.



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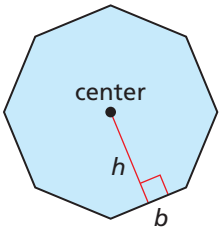

REASONING In Exercises 45 and 46, solve the literal equation for a.

45. $x = \dfrac{a + b + c}{ab}$

46. $y = x\left(\dfrac{ab}{a - b}\right)$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression. *(Skills Review Handbook)*

47. $15 - 5 + 5^2$ **48.** $18 \cdot 2 - 4^2 \div 8$ **49.** $3^3 + 12 \div 3 \cdot 5$ **50.** $2^5(5 - 6) + 9 \div 3$

Solve the equation. Graph the solutions, if possible. *(Section 1.4)*

51. $|x - 3| + 4 = 9$ **52.** $|3y - 12| - 7 = 2$ **53.** $2|2r + 4| = -16$ **54.** $-4|s + 9| = -24$